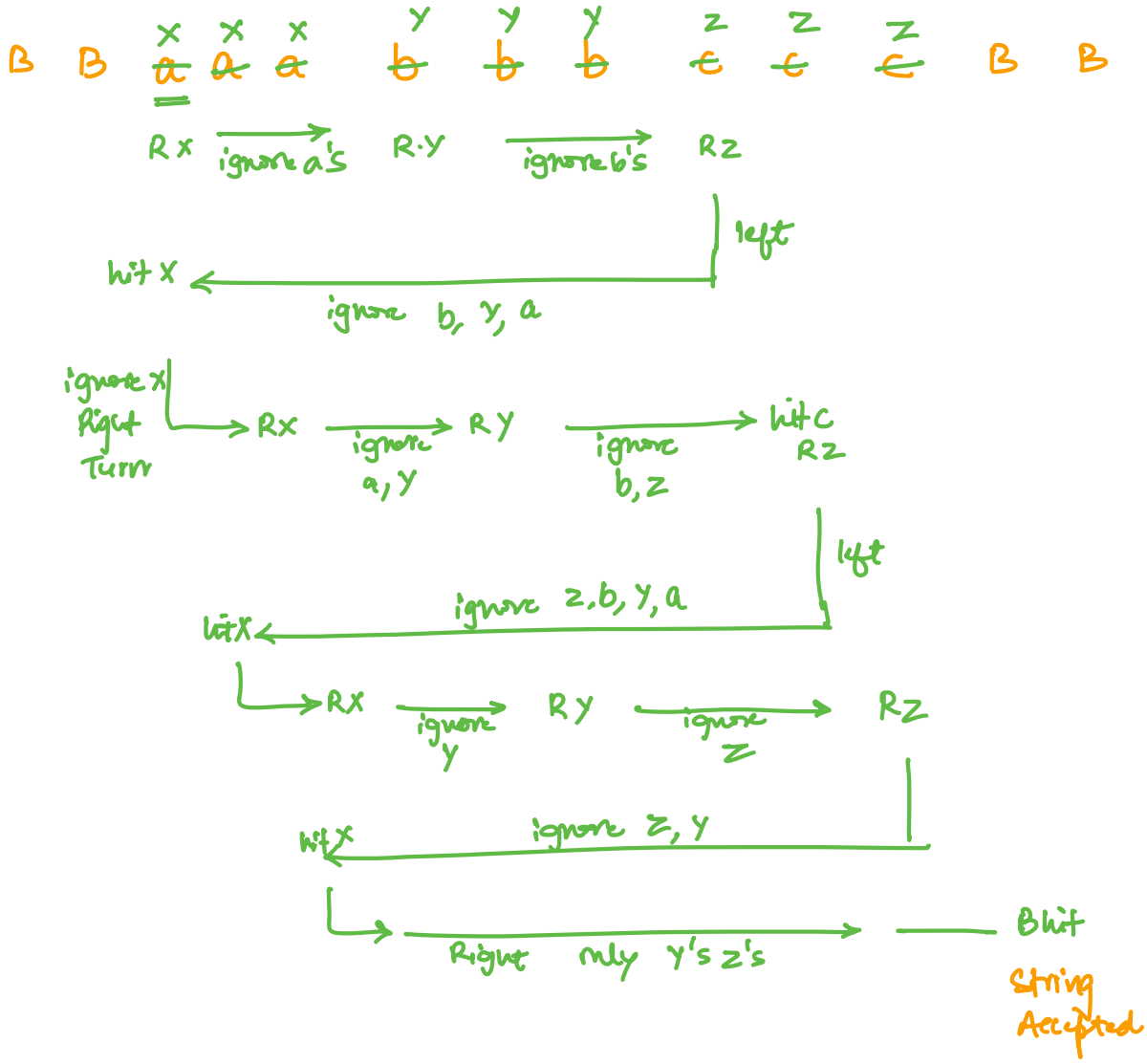
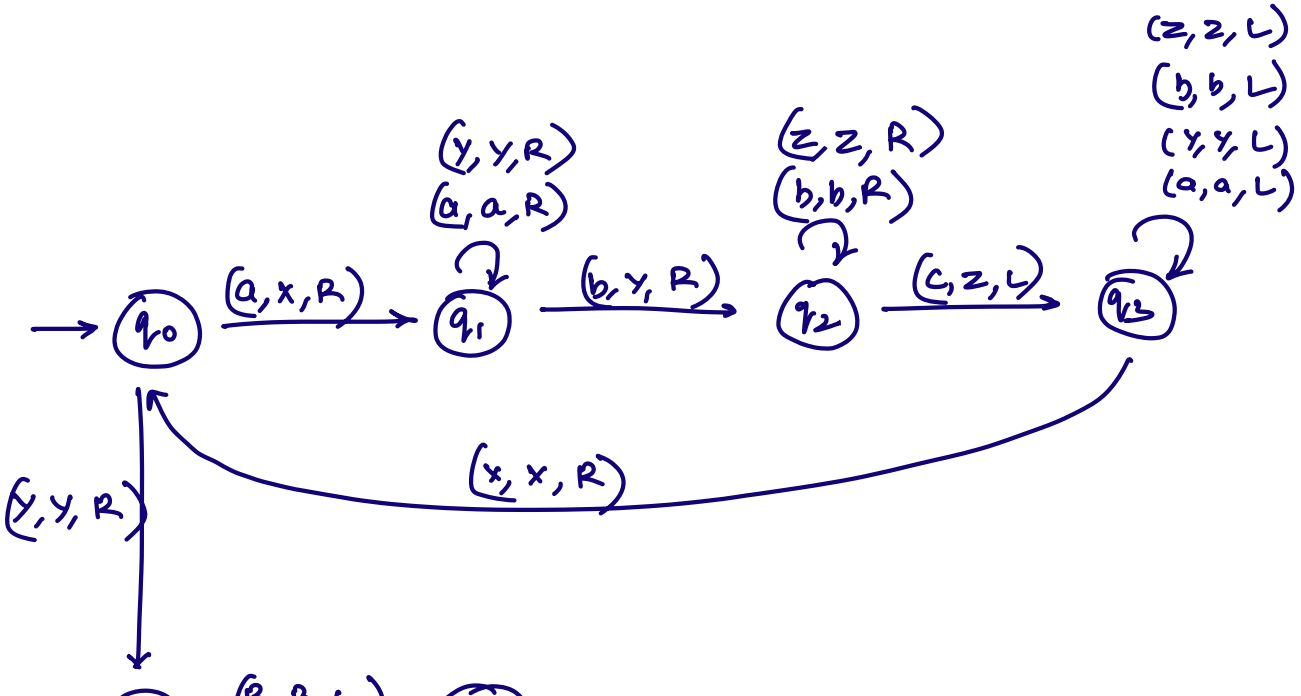
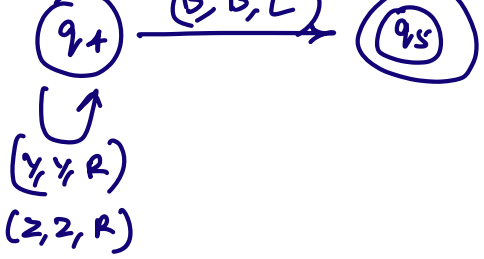
Eg: $a^n b^n c^n \mid n \geq 1$







Example: a a b b b c c
X X Y Y Z Z

q_4 state
b transition ?

Eg: a a b b c c c
X X Y Y Z Z

q_4 state
c transition ?

Eg: bb aa cc

q_0 state
b transition ?

Eg: aaccbb

q_1 state
c ?

Transitions:

$$\delta(q_0, a) = (q_1, X, R)$$
$$\delta(q_0, Y) = (q_4, Y, R)$$
$$\vdots$$

TM can act like as transducer as well as output

acceptor
↳ Accept / Reject $w \in L$ → $a^n b^n$, $a^n b^n c^n$

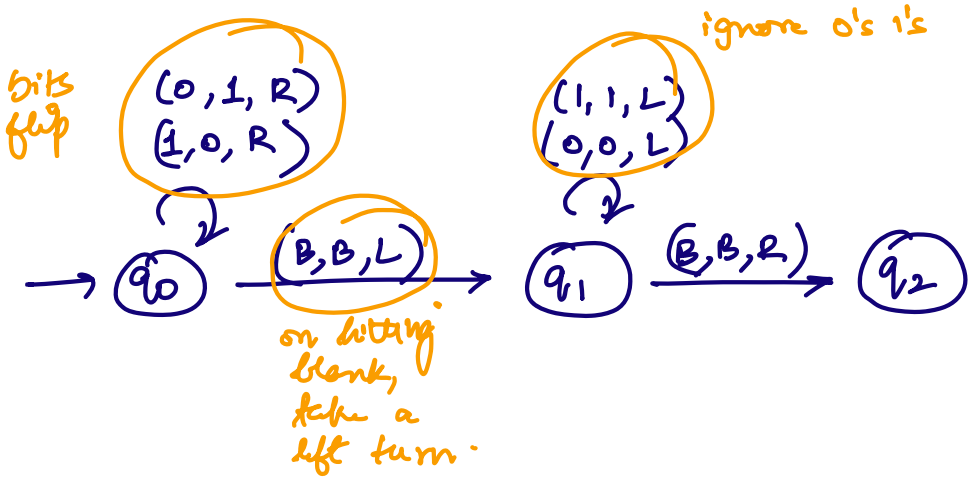
Transducer:

flip the bits (0→1's, 1→0's)

Eg: TM to find 1's complement of a binary no.

Input: 0011

output: 1100

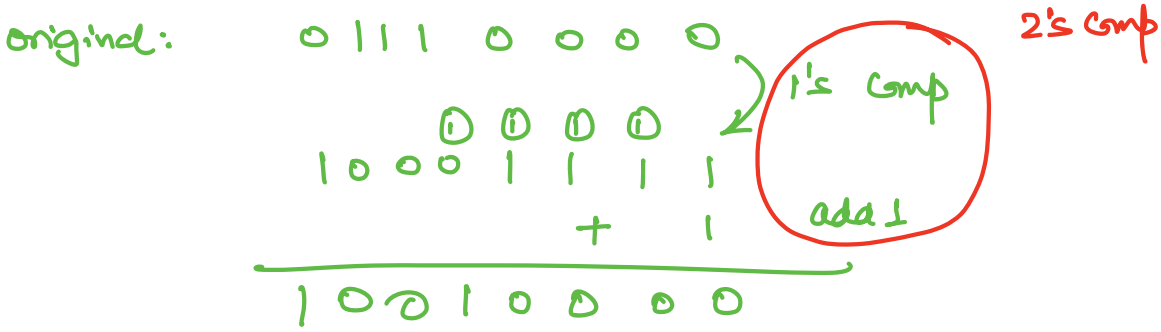


This TM won't say Yes/No. No need of final state

Acceptor → final state

Transducer → no need of final state.

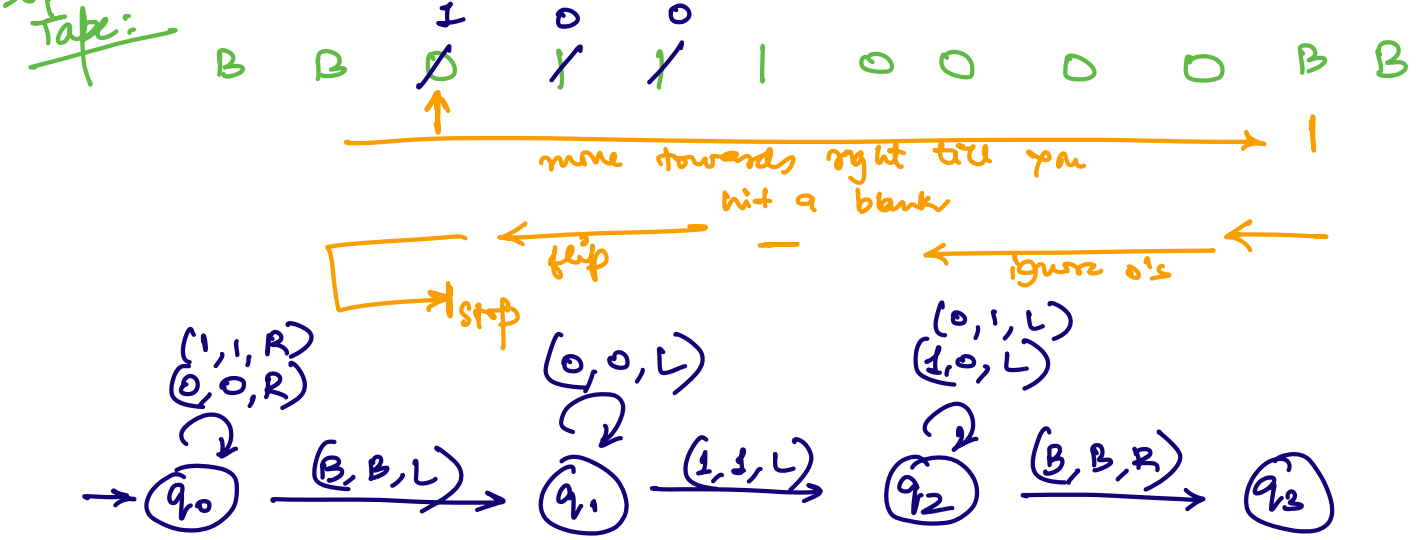
Eg: TM for Computing 2's Complement.



original: 0 1 1 1 0 0 0 0

2's Comp: 1 0 0 1 0 0 0 0

Input



Eg: TM as adder

Unary Number System

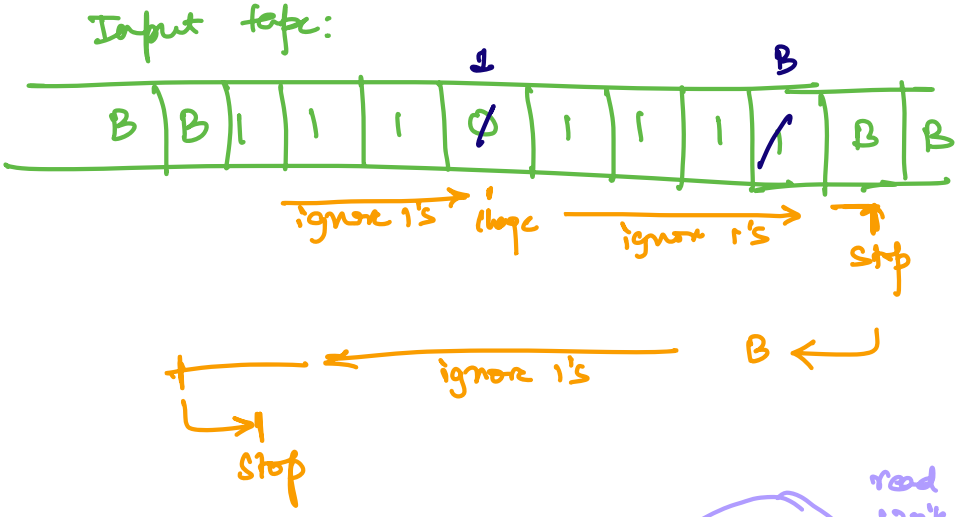
3 → 111 x

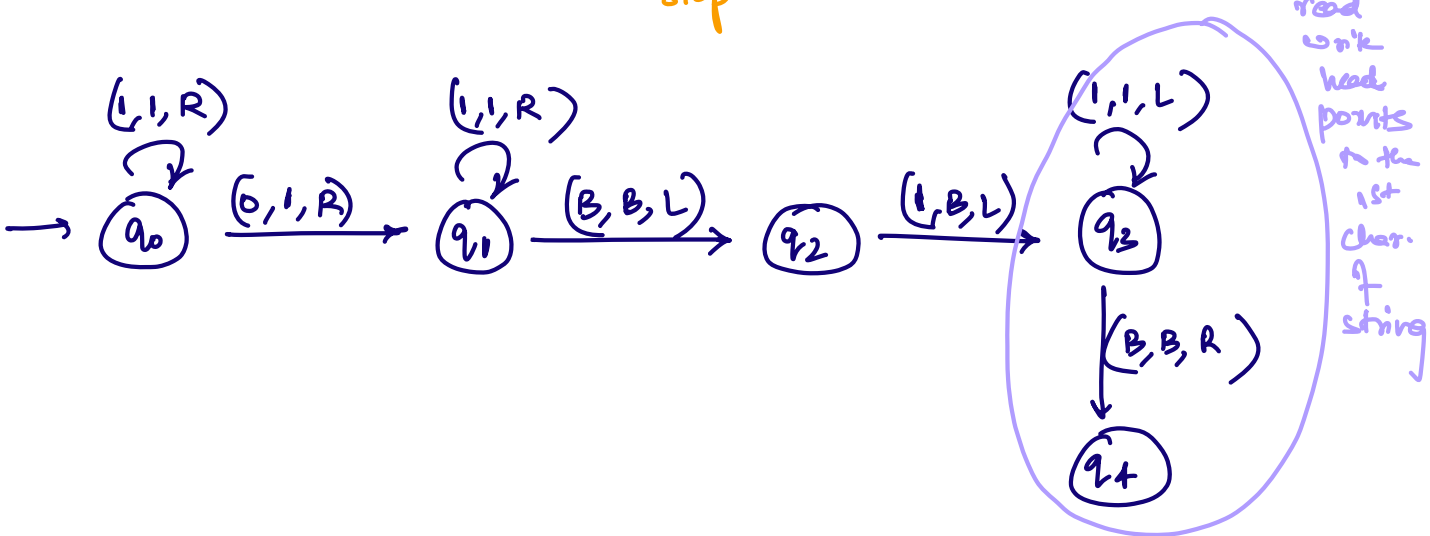
4 → 1111

2 → 11

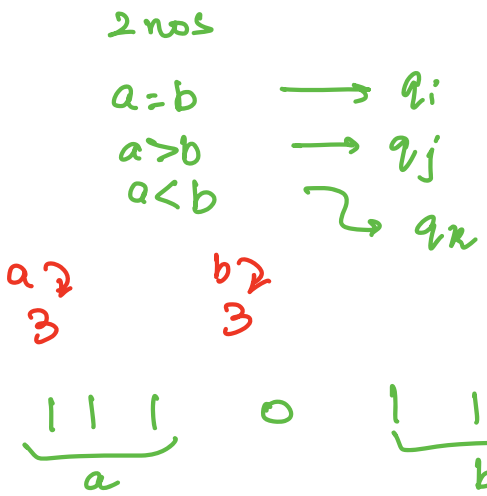
|x| = no

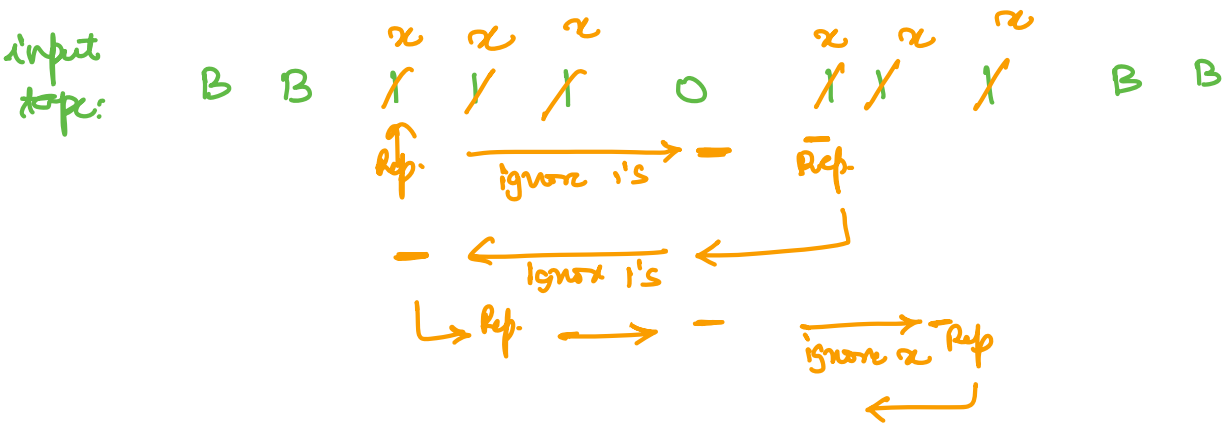
3 + 4
= 7

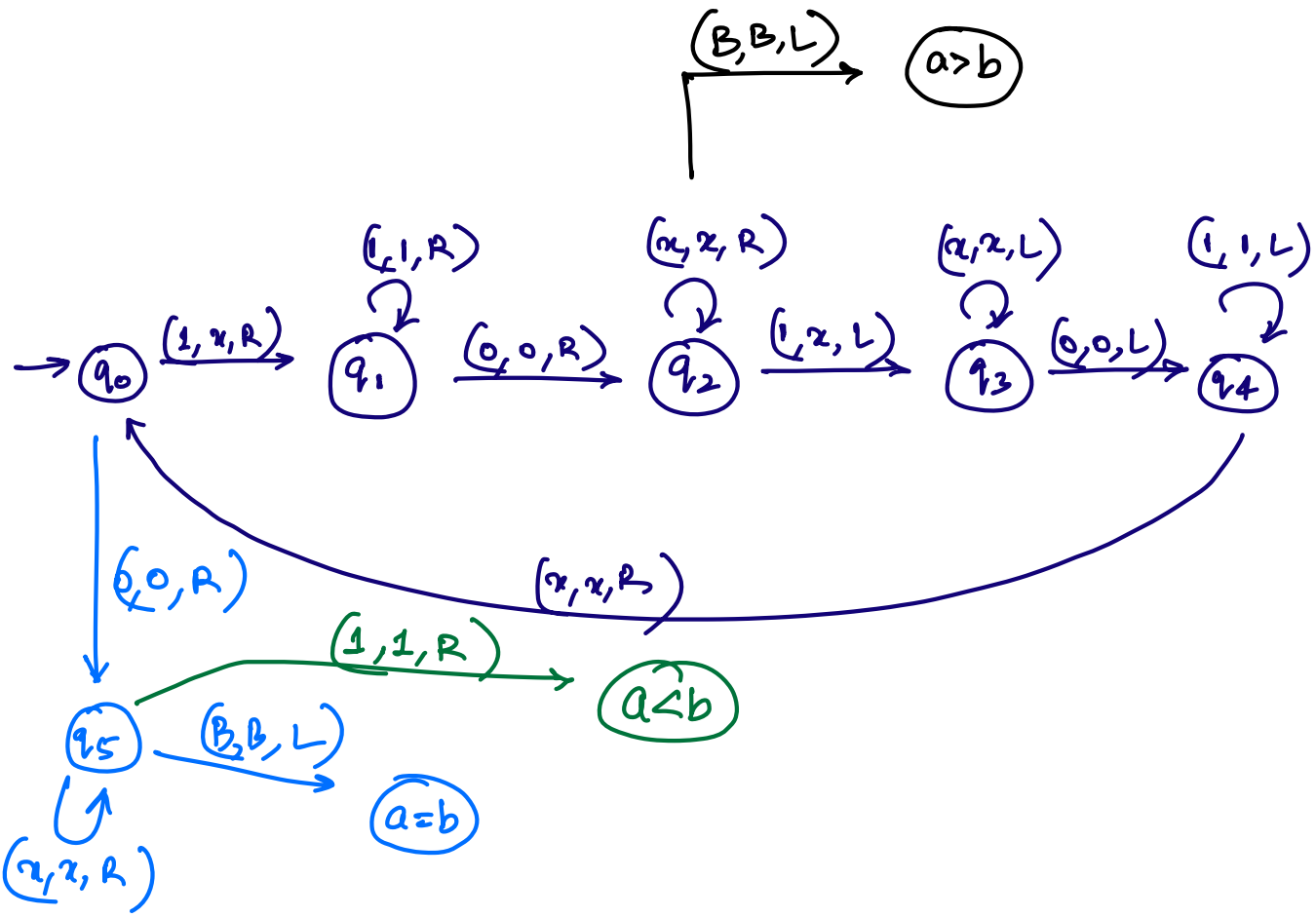




Eg: TM as Comparator







Definition of Standard TM

$M: (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

- Q: Set of states
- Σ: input alphabet
- Γ: Tape alphabet
- δ: Transition fxn
- q_0: Initial state
- B: Blank, $B \in \Gamma$
- F: final state, $F \subseteq Q$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, a=b, a>b, a<b\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, 1, B, x\}$

$\delta(q_0, 1) = (q_1, x, R)$

$\vdots$

$\delta: (Q \times \Gamma) \longrightarrow (Q \times \Gamma \times \{L, R\})$

- Q: state
- Γ: symbol from tape
- Q: new state
- Γ: new symbol
- $\{L, R\}$: left / Right

Properties:

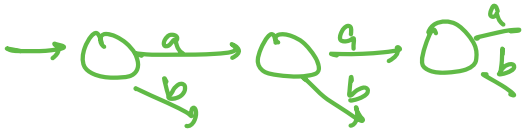
1. Tape is unbounded, δ can take any no. of left & right moves.

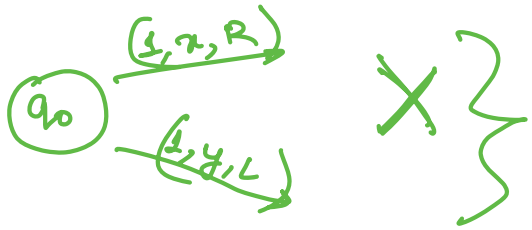
2. TM is deterministic.

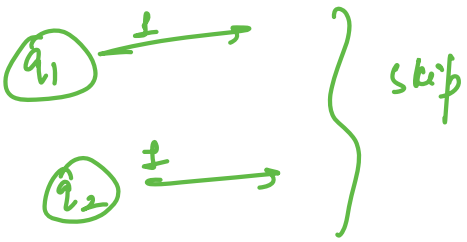
TM is partially deterministic

Fully Deterministic	Partially Deterministic
Every state there should be a transition for every input alphabet.	Not mandatory to have transition for every input alphabet.

DFA.







3. FA + 1 Stack = PDA

PDA + 1 stack = TM

FA + 2 stack = TM

2 Stack: Queue

FA + Queue = TM

↳ Input Tape

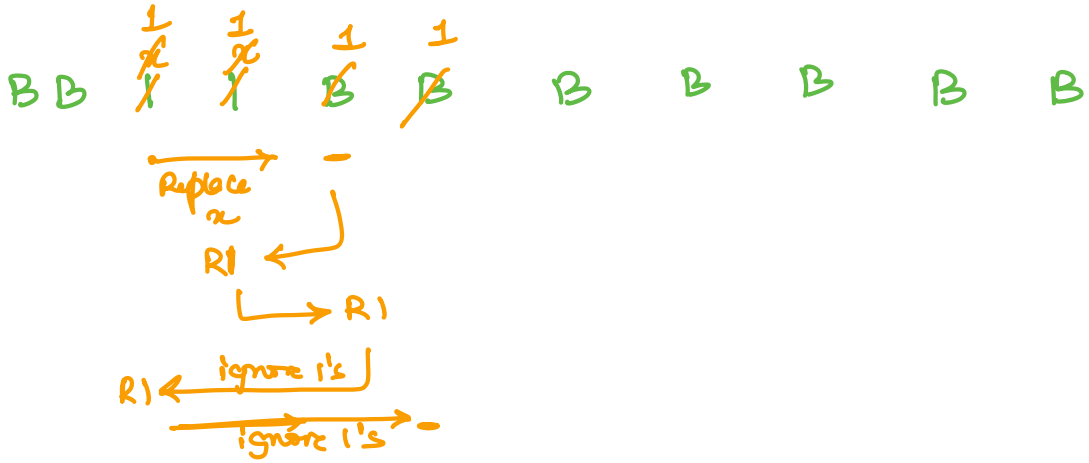
Eg: TM as copier

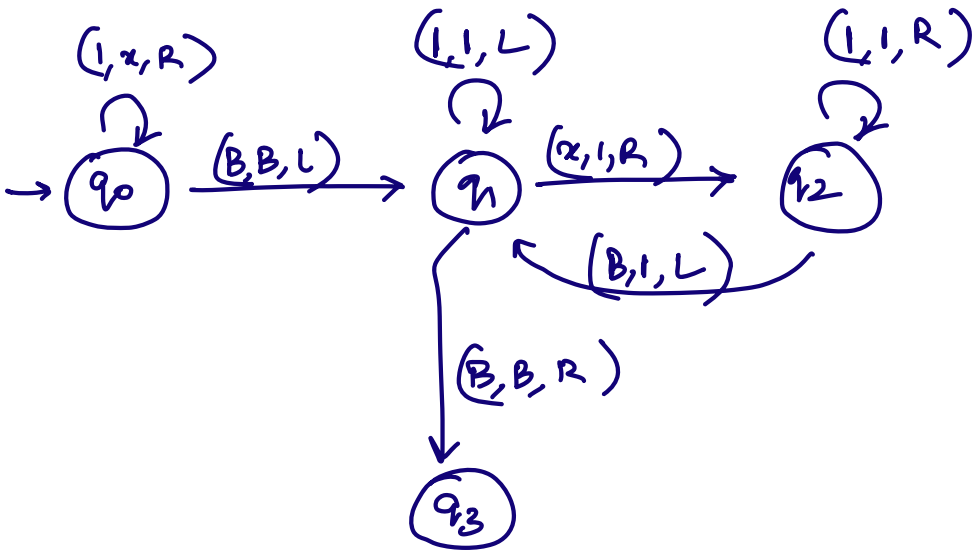
Input:

B	B	1	1	B	B

Output:

B	B	1	1	1	1	B	B





Advantage of copy is $2*3$

copy 3, 2 times

$x*y$

TM can do multiplication

TM is mathematically complete

addition
multiplication
comparison